

Church-Turing Thesis – revisited

Bas van Gijzel

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So first...

A reintroduction to the Church-Turing Thesis and some concepts from Sélinde's talk.

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Alonzo Church (1903-1995)

Church wanted to define the notion of an effectively calculable function.



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An effectively calculable function: a small motivation

- ▶ What is an effectively calculable function?
 - An **informal** concept.
 - Explained as: “calculable by a finite (mechanical) procedure”.
- ▶ Some possible intuitive notions:
 - Functions humans can solve in finite time.
 - Functions that can be constructively defined.

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The Church Thesis

- ▶ Church wanted to define this notion of effectively calculable by using λ -definable functions.
- ▶ After encountering Gödel this was deemed unsatisfactory and the 'thesis' was changed.

New solution:

- ▶ Identify effectively calculable functions with the recursive functions.
- ▶ One identification is done in the context of *calculability in a logic*.
- ▶ Problem: steps of the proof were not proved recursive (or finite and mechanical).

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The Church Thesis by Church

Church's formulation (Church 1936):

We now define the notion, already discussed, of an effectively calculable function of positive integers by identifying it with the notion of a recursive function of positive integers (or a λ -definable function of positive integers). This definition is thought to be justified by the considerations which follow, so far as positive justification can ever be obtained for the selection of formal definition to correspond to an intuitive notion.

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The Church Thesis in short (Sieg)

Due to Sieg (Sieg1997):

The steps of any effective procedure (governing proofs in a system of symbolic logic) must be recursive.

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Alan Turing (1912-1954)

Turing wanted to define the notion of effective calculability or informal computability in terms of an idealized human/computer.



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The Turing Thesis

- ▶ The processes or steps taken by this idealized human are formalised and restrained. Conditions as by Sieg (Sieg2001):
 - (B) (Boundedness) *There is a fixed bound on the number of configurations a computer can immediately recognize.*
 - (L) (Locality) *A computer can change only immediately recognizable (sub-)configurations.*
 - (D) (Determinacy) *The immediately recognizable (sub-)configuration determines uniquely the next computation step (and id).*
- ▶ Turing then argued that number theoretic functions calculable by such a computer are also computable by a Turing machine.

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The Turing Thesis in short

Adaptation of Shagrir (Shagrir2002) which he calls the Gandy-Sieg account:

*Any function that can be computed by a computer, with the previous restrictions (**B**, **L** and **D**), is Turing computable (in the sense of a Turing machine).*

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The two short formulations combined:

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Interpretation of Turing's Thesis(1)

Again the interpretation of Gandy-Sieg:

Any function that can be computed by a computer with the previous restrictions is Turing computable (in the sense of a Turing machine).

- ▶ This formulation uses computers, and thus humans, to formulate the concept effective calculability.
- ▶ Church, as can be seen in his review (Church1937), also probably understood Turing's Thesis as a thesis about computers, not about computing machines. (Hodges2006 and Shagrir2002)

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Interpretation of Turing's Thesis(2)

Again the interpretation of Gandy-Sieg:

Any function that can be computed by a computer with the previous restrictions is Turing computable (in the sense of a Turing machine).

- ▶ This is in contrast to common modern opinions and interpretations of the Church-Turing Thesis.
- ▶ The Church-Turing Thesis in current computer science is often understood in terms of finite **machine** computation in contrast to human computation.

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Interpretation of the (Church-)Turing Thesis

- ▶ The Gandy-Sieg account of an effectively calculable function for the Church-Turing Thesis is an interpretation phrased in computers/idealized human computers, whereas the other view is machine based.
- ▶ So in fact we can formulate two versions of the Church-Turing Thesis (as done in Shagrir2002).

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Human and Machine Church-Turing Thesis

The **human** version of the Church-Turing Thesis:

Any function (of positive integers) that can be computed by a human computer is Turing computable.

The **machine** version of the Church-Turing Thesis:

Any function (of positive integers) that can be computed by a finite machine is Turing computable.

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Variations on the Church-Turing Thesis

- ▶ Quite some variants on the Church-Turing Thesis have been proposed.
- ▶ Also motivated by the two possible interpretations given before.

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Physical Church-Turing Thesis (Shagrir)

- ▶ An abstraction quickly made from human computers and finite machines is the abstraction to physical systems in general.
- ▶ This abstraction also serves as a distinction between proper Church-Turing Thesis as defined before and the **Physical Church-Turing Thesis** (PCTT).
- ▶ This is also called the Strong Church-Turing Thesis expressing that Turing Machines capture all computation, so models of computation more expressive than TMs are impossible.

PCTT as formulated in Shagrir2002:

Any function (of positive integers) that can be computed by a physical system is Turing-computable.

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Physical Church-Turing Thesis (Piccinini)

- ▶ Piccinini makes a further distinction (Piccinini2007) in the PCTT.
- ▶ A modest version which is supposedly equal to the previous formulation by Shagrir.
- ▶ A bold version that possibly broadens the scope of effectively calculable functions.

Modest PCTT:

... the functions whose values are generated by computing mechanisms are Turing-computable.

Bold PCTT:

... the functions whose values are generated (by computation or any other means) by physical systems are Turing-computable.

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- ▶ Copeland has a lot of criticism on the modest version of the Church-Turing Thesis and also already on the machine version of the Church-Turing Thesis.
- ▶ He suggests there could exist physical machines capable of performing computations not computable by Turing Machines.
- ▶ This field is called **hypercomputation** and is still speculative.

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The Extended Church-Turing Thesis (1)

- ▶ Formulated early by Parberry in 1986 using a parallel computation thesis.
- ▶ Yao (Yao2003) calls it an extension of the classical Church-Turing Thesis, Extended Church-Turing Thesis.
- ▶ Beside the standard thesis, the Turing machine is also deemed as efficient as any computing device can be.

Extended Church-Turing Thesis (adaptation of Yao2003):

Any effectively calculable function is Turing computable and efficiently computable using Turing machines.

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The Extended Church-Turing Thesis (2)

- ▶ The Extended Church-Turing Thesis can be used as an interesting starting point for discussion.
- ▶ Discuss physical realisation of computing machines that are capable of solving some problems much faster than the standard computational model.
- ▶ Does not immediately go into the realm of quantum mechanics.

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- ▶ Quantum Complexity-Theoretic Church-Turing Thesis was formulated by Bernstein and Vazirani in 1997.
- ▶ A variant formulation of the Church-Turing Thesis closely related to the Extended Church-Turing Thesis.
 - Would invalidate the extended Church-Turing Thesis because the quantum model of computation is more efficient than a normal Turing machine.

Quantum Complexity-Theoretic Church-Turing Thesis:

A quantum Turing machine can efficiently simulate any realistic model of computation.

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Summary

- ▶ The Church-Turing Thesis can be described as a thesis concerning computers or finite machines.
- ▶ Terminology, implications and exact meaning of the Church-Turing Thesis are still discussed.
- ▶ Variations can put constraints or give more freedom on the physicality and efficiency of the computers/computors.

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Further reading

Some other interesting topics not discussed due to a lack of time:

- ▶ Interactive Turing machines (possibly with advice).
- ▶ All knowing entities that can be used in computations called oracles.
- ▶ Gandy machines, a concept relating to physical machines.
- ▶ Super-recursive algorithms, a class of algorithms that compute more than Turing machines.

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