

Do mathematical theorems like Gödel's show that computers are intrinsically limited?

Bas van Gijzel

May 17, 2010

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions



Universiteit Utrecht

Slides and talk in English.

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions



Outline

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Outline

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions



Outline

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Not one but two

Not Gödel's incompleteness theorem but Gödel's
incompleteness theorems!

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

First incompleteness theorem

The first incompleteness theorem (Gödel-Rosser).

Any *consistent* formal system S within which a certain amount of elementary arithmetic can be carried out is incomplete with regard to statements of elementary arithmetic: there are such statements which can neither be proved, nor disproved in S .

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions



Universiteit Utrecht

Important concepts

- ▶ Elementary arithmetic
- ▶ Formal system
- ▶ Proved/disproved
- ▶ Consistency of a formal system
- ▶ Completeness

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions



Crash course in logic concepts



Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions



Universiteit Utrecht

Formal system

- ▶ Formal system: axioms, inference rules.
- ▶ Propositional logic:
- ▶ Axioms:
 - $\phi \rightarrow (\psi \rightarrow \phi)$
 - $(\phi \rightarrow (\psi \rightarrow \chi)) \rightarrow ((\phi \rightarrow \psi) \rightarrow (\phi \rightarrow \chi))$
 - $(\neg\phi \rightarrow \neg\psi) \rightarrow (\psi \rightarrow \phi)$
- ▶ Inference rule(s):
 - Modus ponens: $\phi \rightarrow \psi, \phi \vdash \psi$

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions



Universiteit Utrecht

Formal system

- ▶ Formal system: axioms, inference rules.
- ▶ Propositional logic:
- ▶ **Axioms:**
 - $\phi \rightarrow (\psi \rightarrow \phi)$
 - $(\phi \rightarrow (\psi \rightarrow \chi)) \rightarrow ((\phi \rightarrow \psi) \rightarrow (\phi \rightarrow \chi))$
 - $(\neg\phi \rightarrow \neg\psi) \rightarrow (\psi \rightarrow \phi)$
- ▶ **Inference rule(s):**
 - Modus ponens: $\phi \rightarrow \psi, \phi \vdash \psi$

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions



Formal system

- ▶ Formal system: axioms, inference rules.
- ▶ Propositional logic:
- ▶ **Axioms:**
 - $\phi \rightarrow (\psi \rightarrow \phi)$
 - $(\phi \rightarrow (\psi \rightarrow \chi)) \rightarrow ((\phi \rightarrow \psi) \rightarrow (\phi \rightarrow \chi))$
 - $(\neg\phi \rightarrow \neg\psi) \rightarrow (\psi \rightarrow \phi)$
- ▶ **Inference rule(s):**
 - Modus ponens: $\phi \rightarrow \psi, \phi \vdash \psi$

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Elementary arithmetic

Intuitively **at least** the following:

- ▶ Natural numbers: $0, 1, 2, \dots$
 - Using 0 and a successor function S .
- ▶ Addition and multiplication.
- ▶ Induction principle.

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Elementary arithmetic

Intuitively **at least** the following:

- ▶ Natural numbers: $0, 1, 2, \dots$
 - Using 0 and a successor function S .
- ▶ Addition and multiplication.
- ▶ Induction principle.

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Elementary arithmetic

Intuitively **at least** the following:

- ▶ Natural numbers: $0, 1, 2, \dots$
 - Using 0 and a successor function S .
- ▶ Addition and multiplication.
- ▶ Induction principle.

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Elementary arithmetic

Intuitively **at least** the following:

- ▶ Natural numbers: $0, 1, 2, \dots$
 - Using 0 and a successor function S .
- ▶ Addition and multiplication.
- ▶ Induction principle.

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Statements of a system

- ▶ A statement is a logical formula, for instance $p \vee \neg p$.
- ▶ A statement ϕ that is provable in S , is denoted as $S \vdash \phi$.

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Statements of a system

- ▶ A statement is a logical formula, for instance $p \vee \neg p$.
- ▶ A statement ϕ that is provable in S , is denoted as $S \vdash \phi$.

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Consistency of a formal system

- ▶ Intuitively: a system does not derive nonsense.
- ▶ A system S is consistent iff $S \not\vdash \perp$.
- ▶ Or, a system cannot simultaneously derive $S \vdash \phi$ and $S \vdash \neg\phi$
- ▶ More to be said in the discussion...

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Consistency of a formal system

- ▶ Intuitively: a system does not derive nonsense.
- ▶ A system S is consistent iff $S \not\vdash \perp$.
- ▶ Or, a system cannot simultaneously derive $S \vdash \phi$ and $S \vdash \neg\phi$
- ▶ More to be said in the discussion...

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Consistency of a formal system

- ▶ Intuitively: a system does not derive nonsense.
- ▶ A system S is consistent iff $S \not\vdash \perp$.
- ▶ Or, a system cannot simultaneously derive $S \vdash \phi$ and $S \vdash \neg\phi$
- ▶ More to be said in the discussion...

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Consistency of a formal system

- ▶ Intuitively: a system does not derive nonsense.
- ▶ A system S is consistent iff $S \not\vdash \perp$.
- ▶ Or, a system cannot simultaneously derive $S \vdash \phi$ and $S \vdash \neg\phi$
- ▶ More to be said in the discussion...

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

(Negation) completeness of a theory

- ▶ Intuitively: All statements are either true or untrue and can be proved so.
- ▶ Very strong property!
- ▶ System T is complete iff for every sentence ϕ : $T \vdash \phi$ or $T \vdash \neg\phi$.
- ▶ **Not:** every true formula can be proved.
(Completeness of FOL)
- ▶ Maximal consistent

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions



Universiteit Utrecht

(Negation) completeness of a theory

- ▶ Intuitively: All statements are either true or untrue and can be proved so.
- ▶ Very strong property!
- ▶ System T is complete iff for every sentence ϕ : $T \vdash \phi$ or $T \vdash \neg\phi$.
- ▶ **Not:** every true formula can be proved.
(Completeness of FOL)
- ▶ Maximal consistent

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions



Universiteit Utrecht

(Negation) completeness of a theory

- ▶ Intuitively: All statements are either true or untrue and can be proved so.
- ▶ Very strong property!
- ▶ System T is complete iff for every sentence ϕ : $T \vdash \phi$ or $T \vdash \neg\phi$.
- ▶ **Not:** every true formula can be proved.
(Completeness of FOL)
- ▶ Maximal consistent

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions



Universiteit Utrecht

Incompleteness

- ▶ System does not have good enough inference rules.
- ▶ There is a sentence for T, ϕ , for which $T \vdash \phi$ and $T \vdash \neg\phi$.
- ▶ Derivation is undecidable.
- ▶ Or: sentences of the system are not recursively enumerable.

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Incompleteness

- ▶ System does not have good enough inference rules.
- ▶ There is a sentence for T, ϕ , for which $T \vdash \phi$ and $T \not\vdash \neg\phi$.
- ▶ Derivation is undecidable.
- ▶ Or: sentences of the system are not recursively enumerable.

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Incompleteness

- ▶ System does not have good enough inference rules.
- ▶ There is a sentence for T, ϕ , for which $T \vdash \phi$ and $T \not\vdash \neg\phi$.
- ▶ Derivation is undecidable.
- ▶ Or: sentences of the system are not recursively enumerable.

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Incompleteness

- ▶ System does not have good enough inference rules.
- ▶ There is a sentence for T, ϕ , for which $T \vdash \phi$ and $T \not\vdash \neg\phi$.
- ▶ Derivation is undecidable.
- ▶ Or: sentences of the system are not recursively enumerable.

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

End of crash course



Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions



Universiteit Utrecht

First incompleteness theorem (again)

The first incompleteness theorem (Gödel-Rosser).

Any *consistent* formal system S within which a certain amount of elementary arithmetic can be carried out is incomplete with regard to statements of elementary arithmetic: there are such statements which can neither be proved, nor disproved in S .

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions



Universiteit Utrecht

Summary

So what does Gödel's first Incompleteness Theorem say?

- ▶ About **axiomatised formal theories** of arithmetic.
- ▶ In short, arithmetical truth isn't provability in some single axiomatisable system.
- ▶ If T is consistent: $\exists G_T : T \vdash G_T$ and $T \not\vdash \neg G_T$.
- ▶ But it also holds that: $Con_T \rightarrow G_T$.

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Summary

So what does Gödel's first Incompleteness Theorem say?

- ▶ About **axiomatised formal theories** of arithmetic.
- ▶ In short, arithmetical truth isn't provability in some single axiomatisable system.
- ▶ If T is consistent: $\exists G_T : T \vdash G_T$ and $T \not\vdash \neg G_T$.
- ▶ But it also holds that: $Con_T \rightarrow G_T$.

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Outline

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Possible questions (1)

Are there unprovable truths?

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Possible questions (1)

Are there unprovable truths?

▶ $S \not\vdash A? \quad S \cup A \vdash A!$

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Possible questions (2)

Lucas argument:

Gödel's theorem states that in any consistent system which is strong enough to produce simple arithmetic there are formulas which cannot be proved in the system, but which we can see to be true.

- ▶ We have to prove consistency for that system!
- ▶ Humans cannot prove formal systems consistent in general.
- ▶ “Going out of the system” is not possible in general. (Goldbach's conjecture)

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions



Universiteit Utrecht

Possible questions (2)

Lucas argument:

Gödel's theorem states that in any consistent system which is strong enough to produce simple arithmetic there are formulas which cannot be proved in the system, but which we can see to be true.

- ▶ We have to prove consistency for that system!
- ▶ Humans cannot prove formal systems consistent in general.
- ▶ “Going out of the system” is not possible in general.
(Goldbach's conjecture)

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Possible questions (2)

Lucas argument:

Gödel's theorem states that in any consistent system which is strong enough to produce simple arithmetic there are formulas which cannot be proved in the system, but which we can see to be true.

- ▶ We have to prove consistency for that system!
- ▶ Humans cannot prove formal systems consistent in general.
- ▶ “Going out of the system” is not possible in general.
(Goldbach's conjecture)

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

(Negation) completeness of a theory

The ... claims to give all answers/claims to be a complete system. By Gödel's incompleteness theorems this cannot be true!

- ▶ bible.
- ▶ law.
- ▶ system of propositional logic.

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions



(Negation) completeness of a theory

The ... claims to give all answers/claims to be a complete system. By Gödel's incompleteness theorems this cannot be true!

- ▶ bible.
- ▶ law.
- ▶ system of propositional logic.

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions



Outline

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Outline

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Second incompleteness theorem

The second incompleteness theorem (Gödel).

For any consistent formal system S within which a certain amount of elementary arithmetic can be carried out, the consistency of S cannot be proved in S itself.

Gödel's First
Incompleteness
Theorem

[Introduction](#)

[Discussion](#)

Gödel's Second
Incompleteness
Theorem

[Introduction](#)

[Discussion](#)

Answer to the
question

[More questions](#)



Universiteit Utrecht

Amount of elementary arithmetics

Different amount of arithmetics!

Gödel numbering of sentences.

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Amount of elementary arithmetics

Different amount of arithmetics!

Gödel numbering of sentences.

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Summary

So what do Gödel's Second Incompleteness Theorems say?

- ▶ About **axiomatised formal theories** of arithmetic.
- ▶ Certain amount of arithmetic.
- ▶ Such a system cannot prove its own consistency.

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Summary

So what do Gödel's Second Incompleteness Theorems say?

- ▶ About **axiomatised formal theories** of arithmetic.
- ▶ Certain amount of arithmetic.
- ▶ Such a system cannot prove its own consistency.

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Universiteit Utrecht

Outline

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



Outline

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

**Answer to the
question**

More questions



What we can claim

- ▶ Arithmetical truth isn't provability in some single axiomatisable system.
- ▶ No super foundation of mathematics that is complete. (Principia Mathematica)

Gödel's First Incompleteness Theorem

[Introduction](#)

[Discussion](#)

Gödel's Second Incompleteness Theorem

[Introduction](#)

[Discussion](#)

Answer to the question

[More questions](#)



What we can claim

- ▶ Arithmetical truth isn't provability in some single axiomatisable system.
- ▶ No super foundation of mathematics that is complete. (Principia Mathematica)

Gödel's First Incompleteness Theorem

[Introduction](#)

[Discussion](#)

Gödel's Second Incompleteness Theorem

[Introduction](#)

[Discussion](#)

Answer to the question

[More questions](#)



What we should not claim

- ▶ By Gödel's theorems I hereby pronounce...
 - machines less powerful than humans.
 - formal systems useless.
 - logic as an attempt to formalise AI useless.

Gödel's First Incompleteness Theorem

[Introduction](#)

[Discussion](#)

Gödel's Second Incompleteness Theorem

[Introduction](#)

[Discussion](#)

Answer to the question

[More questions](#)



Do mathematical theorems like Gödel's show that computers are intrinsically limited?

Do mathematical theorems like Gödel's show that computers are intrinsically limited?

Gödel's First Incompleteness Theorem

[Introduction](#)

[Discussion](#)

Gödel's Second Incompleteness Theorem

[Introduction](#)

[Discussion](#)

Answer to the question

[More questions](#)



Universiteit Utrecht

Do mathematical theorems like Gödel's show that computers are intrinsically limited?

Do mathematical theorems like Gödel's show that computers are intrinsically limited?

No!

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions



Outline

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions

Gödel's First
Incompleteness
Theorem

Introduction

Discussion

Gödel's Second
Incompleteness
Theorem

Introduction

Discussion

Answer to the
question

More questions



What we can ask(1)

- ▶ Are humans formal systems?
- ▶ Are humans “complete”?
- ▶ Do we even care?
- ▶ An infinitude of human formal systems?

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions



What we can ask(1)

- ▶ Are humans formal systems?
- ▶ Are humans “complete”?
- ▶ Do we even care?
- ▶ An infinitude of human formal systems?

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions



What we can ask(1)

- ▶ Are humans formal systems?
- ▶ Are humans “complete”?
- ▶ Do we even care?
- ▶ An infinitude of human formal systems?

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions



What we can ask(2)

- ▶ **What can humans effectively calculate?**
- ▶ Related: Church-Turing thesis and its variants.
- ▶ Are mathematics useful if they have so little practical consequences?

Gödel's First Incompleteness Theorem

Introduction

Discussion

Gödel's Second Incompleteness Theorem

Introduction

Discussion

Answer to the question

More questions



Universiteit Utrecht

What we can ask(2)

- ▶ What can humans effectively calculate?
- ▶ Related: Church-Turing thesis and its variants.
- ▶ Are mathematics useful if they have so little practical consequences?

Gödel's First Incompleteness Theorem

[Introduction](#)

[Discussion](#)

Gödel's Second Incompleteness Theorem

[Introduction](#)

[Discussion](#)

[Answer to the question](#)

[More questions](#)



What we can ask(2)

- ▶ What can humans effectively calculate?
- ▶ Related: Church-Turing thesis and its variants.
- ▶ Are mathematics useful if they have so little practical consequences?

Gödel's First
Incompleteness
Theorem

[Introduction](#)

[Discussion](#)

Gödel's Second
Incompleteness
Theorem

[Introduction](#)

[Discussion](#)

[Answer to the
question](#)

[More questions](#)



Universiteit Utrecht